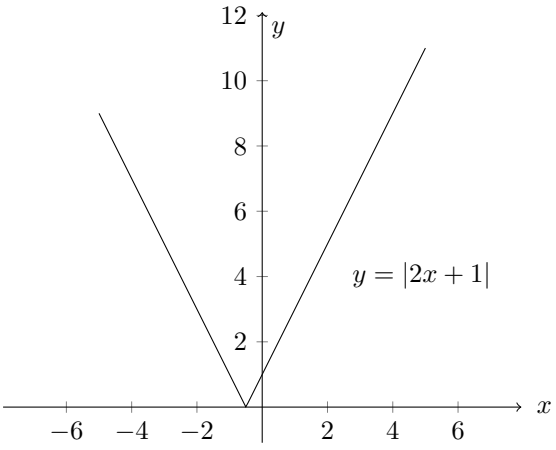
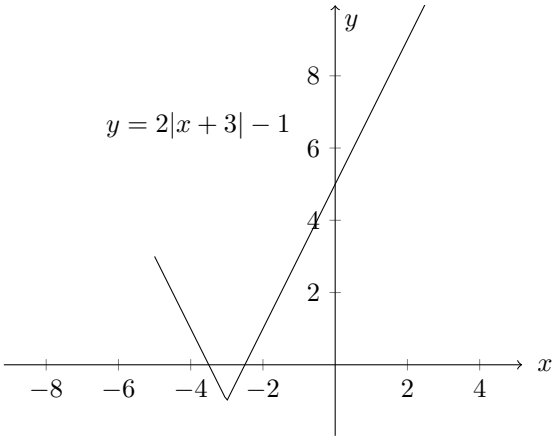
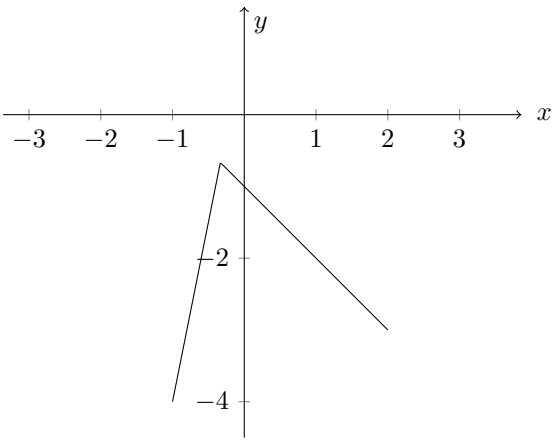
P3 Exercise Solution

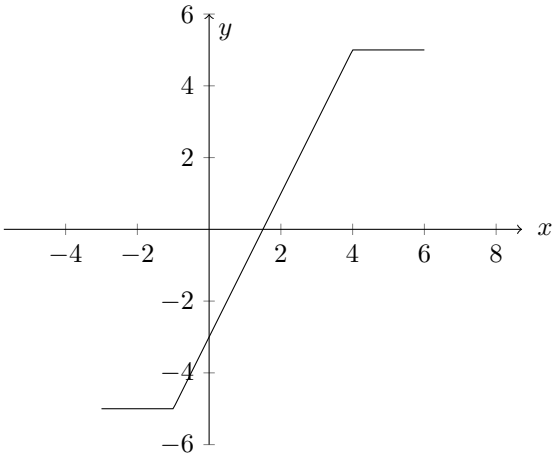
updated: January 12, 2022

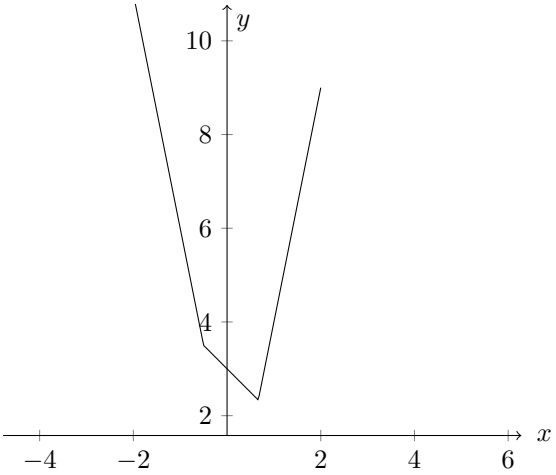
Exercise 1

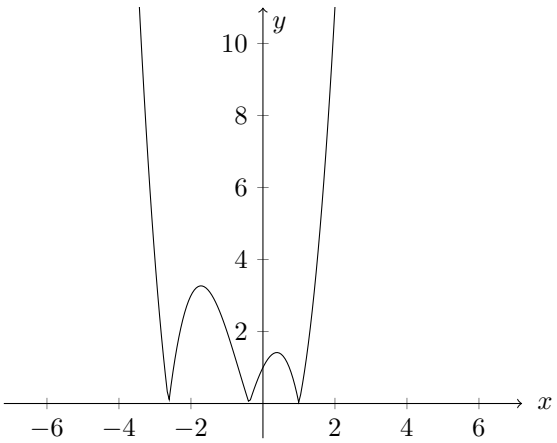


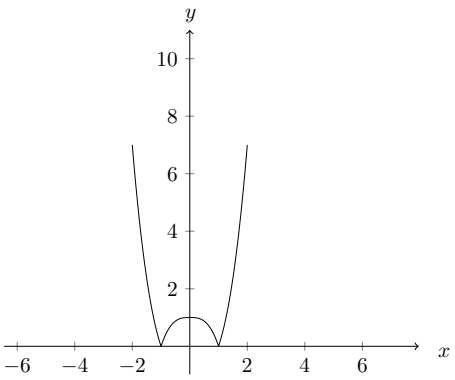


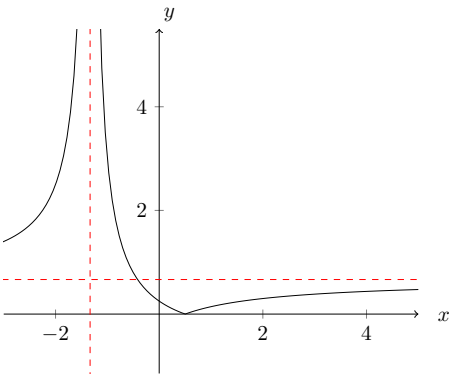


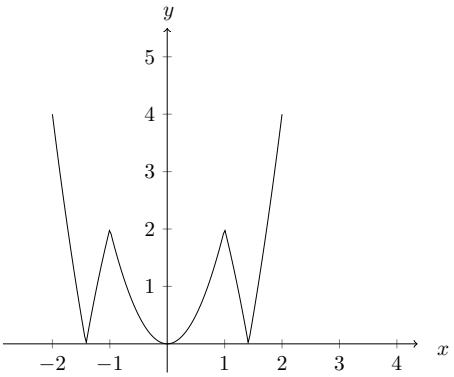


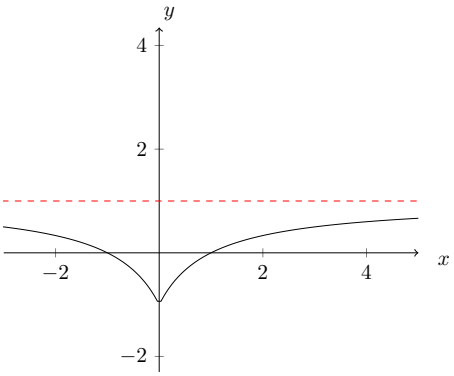












Exercise 2

1. 3 or -2
2. $\frac{2}{3}$ or -5
3. $\frac{3}{2}$
4. 2 or $-\frac{4}{3}$
5. 2
6. $-\frac{8}{7}$ or $\frac{12}{5}$
7. -2, $\frac{7}{3}$, 1, $-\frac{4}{3}$
8. -3, -1, 3,1
9. 1 or -1
10. 2 or -2

Exercise 3

1. $(-\infty, -\frac{1}{2}) \cup (1, \infty)$
2. $[-3, 1]$

3. $(-\frac{3}{2}, 4)$
4. $[1, \infty]$
5. $(-\infty, -2) \cup (\frac{1-\sqrt{17}}{2}, 1) \cup (\frac{1+\sqrt{17}}{2}, \infty)$
6. $(-\infty, \frac{5}{3}) \cup (5, \infty)$
7. $(-3, -1)$
8. $(-\infty, -\frac{3}{4}] \cup [\frac{7}{4}, \infty)$
9. $(-\infty, \frac{5-\sqrt{17}}{2}) \cup (2, 3) \cup (\frac{5+\sqrt{17}}{2}, \infty)$
10. $(-\infty, -\frac{7}{2}) \cup (-2, 0) \cup (0, 2) \cup (\frac{7}{2}, \infty)$

Exercise 4

1. Polynomial: $\frac{(x+1)(x+2)\cdot(x+n)}{n!}$, degree: n, leading term: $\frac{1}{n!}$, constant term 1.
2. $6x^4 + 3x^3 - 18x^2 - 9x - 2$
3. $A = 2$, $B = 3$, $C = -5$.
4. (a) m or n
   (b) m or n
   (c) $m + n$.

Exercise 5

1. quotient: $2x^2 - 3x + 4$, remainder: -13
2. quotient: $3x^2 + 4x + 1$, remainder: $9x - 7$
3. quotient: $x^4 - x^3 + x - 1$, remainder: 2
4. quotient: $x^{n-1} + kx^{n-2} + k^2x^{n-3} + \cdot + k^{n-1}x + k^n$, remainder: k^{n+1}

Exercise 6

1. (a) 302
   (b) $-\frac{1}{3}$
   (c) $2\frac{3}{4}$
2. $a = -\frac{5}{3}$, $b = \frac{1}{3}$
3. $k = -9$, $(x - 2)(2x + 1)(3x + 1)$
4. $x > 1$
5. (a) $f(1) \neq 0$, and $f(-1) \neq 0$,
   (b) quotient: $x^3 + 3x^2 + 3x + 5$, remainder: $6x + 4$
   (c) omit

(d) -1, $-1+\sqrt{2}$, $-1-\sqrt{2}$.

Exercise 7

1. (a) $1+6x+27x^2+108x^3$
   (b) $1+\frac{2}{3}x-\frac{4}{9}x^2+\frac{40}{81}x^3$
   (c) $2-\frac{1}{4}x-\frac{1}{64}-\frac{1}{512}x^3$
   (d) $\frac{1}{27}-\frac{2}{27}x+\frac{8}{81}x^2-\frac{80}{729}x^3$.
2. (a) $4+x-\frac{1}{16}x^2$
   (b) $2-\frac{1}{32}x-\frac{131}{4096}x^2$
   (c) $\frac{1}{2}+\frac{1}{4}x^2$
   (d) $\frac{1}{2}-\frac{1}{16}x-\frac{117}{256}x^2$.
3. $a=4$, $b=12$, $c=-192$
4. (a) $(-1)^n 2^n$
   (b) $\dfrac{(2n-1)!!}{2^n n!}$
   (c) $\dbinom{|k|+n-1}{n}$.
5. By part 4(c), $x^r : \frac{1}{2}(r+1)(r+2)$; $x^{r-1} : -\frac{1}{2}r(r+1)$; $x^{r-2} : r(r-1)$.
   Add them together to get r^2+1.
   (a) 8
   (b) 12.

Exercise 8

1. $-\frac{1}{x}-\frac{\frac{1}{2}}{x+1}+\frac{\frac{3}{2}}{x-1}$.
2. $x^2-x+3-\frac{\frac{16}{3}}{x+2}+\frac{\frac{1}{3}}{x-1}$.
3. $\frac{\frac{1}{2}}{x+1}-\frac{4}{x+2}+\frac{\frac{9}{2}}{x+3}$.

Exercise 9

1. $\frac{1}{x}-\frac{1}{x-1}+\frac{2}{(x-1)^2}$.
2. $\frac{-\frac{7}{4}}{x}+\frac{\frac{11}{4}}{x-1}+\frac{\frac{3}{2}}{(x-1)^2}$.
3. $x+\frac{\frac{16}{9}}{x-2}+\frac{\frac{11}{9}}{x+2}+\frac{-\frac{1}{3}}{(x+1)^2}$.
4. $\frac{\frac{3}{4}}{(x-1)^2}+\frac{\frac{1}{4}}{(x+1)^2}$.

Exercise 10

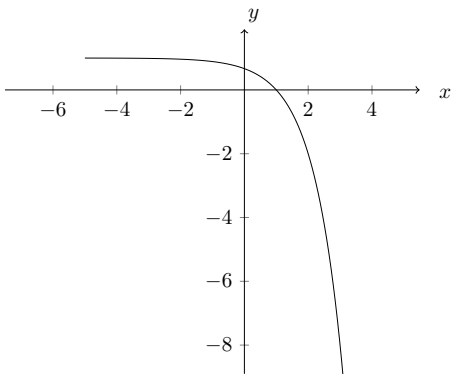
1. $\frac{1}{x} + \frac{-x+1}{x^2+1}$
2. $\frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{x}{x^2+x+1}$
3. $x + 1 + \frac{2}{x-2} + \frac{-3x+1}{2x^2+2x+1}$.
4. $\frac{1}{x-1} + \frac{2}{x+2} + \frac{-2x-3}{x^2-2x+4}$.
5. $\frac{2x-3}{x^2-2x+2} + \frac{2x+3}{x^2+2x+2}$.

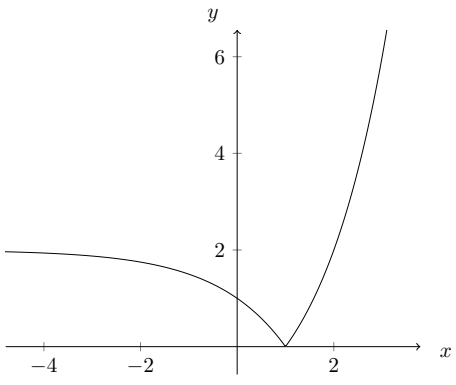
Exercise 11

1. (a) $\frac{1}{x+1} - \frac{6}{x+2} + \frac{8}{x+4}$
   (b) $-\frac{1}{x+1} + \frac{1}{x+2} + \frac{2}{(x+2)^2}$.
   (c) $2 - \frac{2}{x} - \frac{3}{x^2} + \frac{4}{x-1}$
   (d) $\frac{1}{x} - \frac{2}{x-1} + \frac{3x-1}{x^2+1}$
   (e) $1 + \frac{\frac{1}{3}}{x-3} - \frac{\frac{1}{3}}{x+3} - \frac{2}{(x+3)^2}$.
2. (a) $\frac{1}{2} - \frac{3}{4}x + \frac{13}{24}x^2 - \frac{47}{144}x^3$.
   (b) $-1 - \frac{4}{3}x - x^2 - \frac{7}{9}x^3$
   (c) $\frac{5}{2} - \frac{3}{2}x + \frac{5}{4}x^2 - \frac{11}{4}x^3$.
3. $\frac{1}{x-5} - \frac{3}{x+5}$, $f'(x) = -(x-5)^{-2} + 3(x+5)^{-2}$.
4. $1 - \frac{1}{x-1} - \frac{2}{(x-1)^2} + \frac{2}{x-4}$, $f'(x) = (x-1)^{-2} + 4(x-1)^{-3} - 2(x-4)^{-2}$.

Exercise 12

1. $x \in (-\infty, 0) \cup (0, \infty)$, $y \in (0, \infty)$
2. The graphs are as following:





3. $>, <, <, <, <$
4. $m < n$
5. $a > 1$, then $x < -6$
   $a < 1$, then $x > -6$

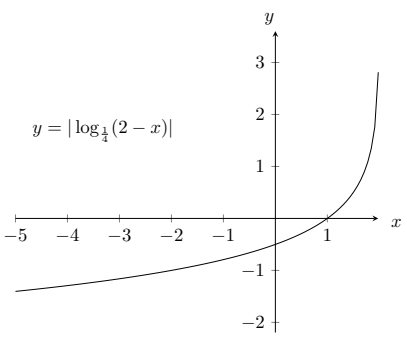
6. (a) $\sqrt{5}$
   (b) 7
   (c) $8\sqrt{5}$ or $-8\sqrt{5}$
7. no
8. 3 or $-\frac{1}{3}$
9. $f(0) = 1$, $f(\frac{1}{p} + \cdots + \frac{1}{p}) = \left[f(\frac{1}{p})\right]^p = f(1) = a$, then $f(\frac{1}{p}) = a^{\frac{1}{p}}$, which is $f(x) = a^x$.

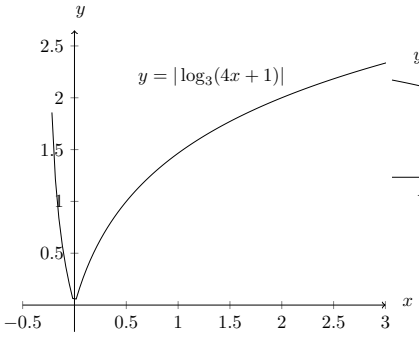
Exercise 13

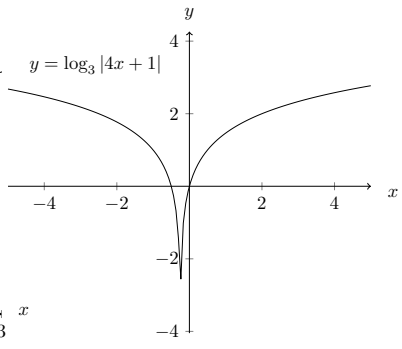
1. 3, -3, $-\frac{7}{4}$, $\frac{37}{2}$, $\frac{5}{4}$
2. $\frac{121}{3}$
3. $\frac{5\ln 2 - \ln 3}{2\ln 3 - 3\ln 2}$.
4. $\frac{\ln 2}{\ln 3 + 2\ln 2} = 0.279$
5. $x = \frac{\ln 5}{\ln 15}$, $y = \frac{\ln 3}{\ln 15}$.
6. $\frac{5}{6} + \log_{10} \frac{3}{2}$
7. (a) $a^{\ln b} = e^{\ln a \ln b} = b^{\ln a}$, then $\left(a^{\ln b}\right)^{\frac{1}{\ln c}} = \left(b^{\ln a}\right)^{\frac{1}{\ln c}}$
   (b) omit
   (c) omit
8. 2
9. 25

Exercise 14

1. $<, >, >, >$
2. The graph are as following:







3. $m > n$
4. (a) $(-1, 1)$
   (b) even.

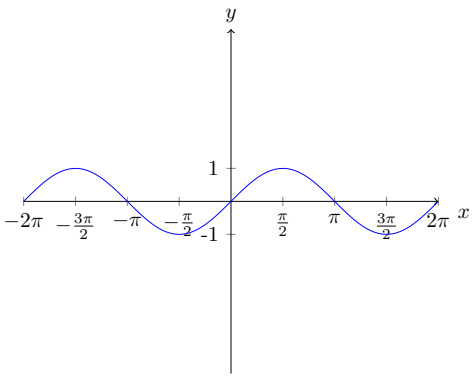
5. $y = a^x - 1$
6. $0 < a < 1$, $0 < x < \log_a(\sqrt{3} - 1)$, $a > 1$, $\log_a(\sqrt{3} - 1) < x < 0$
7. $k = 1$, $\mathbb{R}$
8. $m = n = 5$

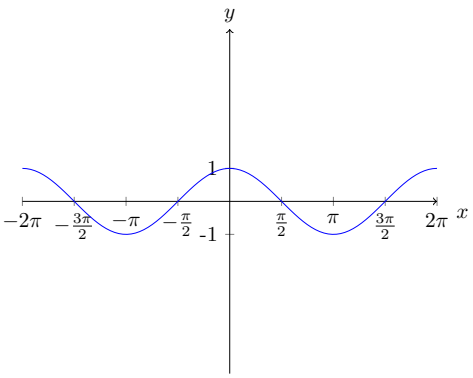
Exercise 15

1. (a) 0.323
   (b) 130
2. $a = \frac{3}{5 \lg 2}$, $b = \frac{5}{2} - \frac{3}{5 \lg 2}$
3. $A = 0.4$, $c = 1.01$, $x = 504$
4. (a) 5
   (b) 1,1,9
5. Consider $f(1) = 0$

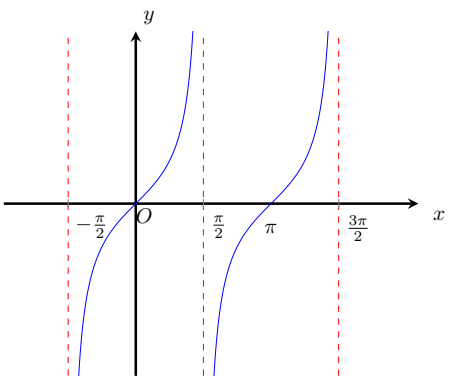
Exercise 16

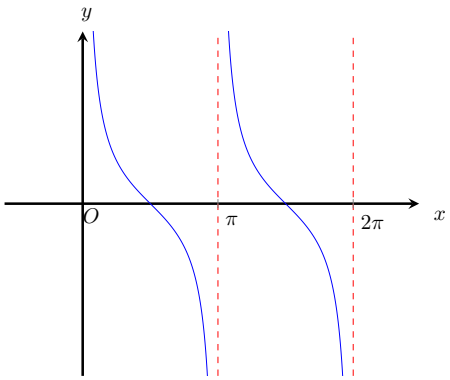
1. The graph of $y = \sin x$ and $y = \cos x$:

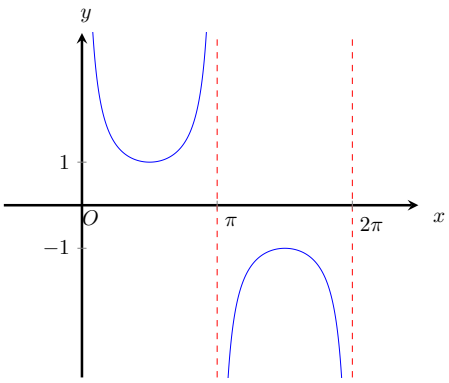


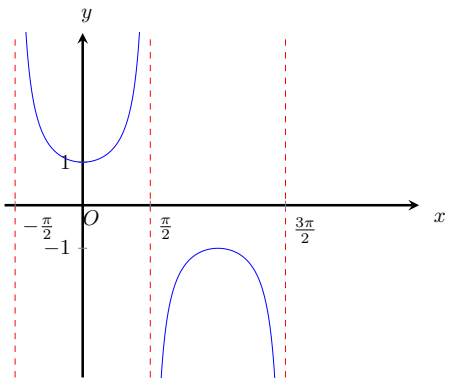


2. The graph of $y = \tan x$ and $y = \cot x$:









3. The graph of $y = \sec x$ and $y = \csc x$:

Exercise 17

1. (a) 0.5
   (b) $\frac{131}{65}$
   (c) $\frac{457}{289}$
   (d) $\frac{3}{5}$
2. omit
3. (a) $73.9°, 286.1°$
   (b) $37.2°, 217.2°, 327.8°, 142.8°$
   (c) $30°, 210°, 150°, 330°$
   (d) $60°, 240°, 120°, 300°$

Exercise 18

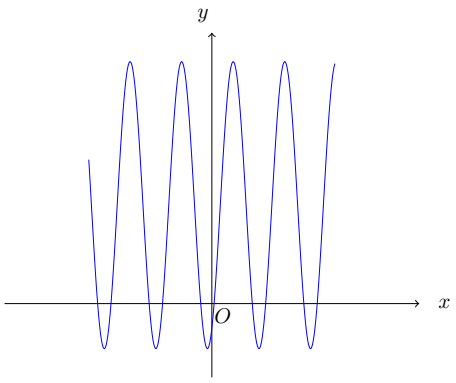
1. $\frac{\sqrt{2}}{4}(\sqrt{3}-1)$
2. $\frac{\sqrt{3}\tan x+1}{\tan x-\sqrt{3}}$
3. $-\frac{56}{65}, \frac{63}{65}, -\frac{16}{63}$
4. $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}$
5. $0.12, 3.26, -3.02, -6.16$
6. $-\frac{59}{72}$
7. $-\frac{4}{3}, -\frac{3}{4}$
8. 0
9. Use $\sin 2\xi = 2\sin\xi\cos\xi$, $\cos\xi = 1 - 2\sin^2\xi$.

Exercise 19

1. $\pm\frac{11}{6}$, $\pm\frac{5}{6}$
2. $-1\pm\sqrt{2}$, $-1+\sqrt{2}$
3. $\frac{\pi}{2}$, 5.44, 2.29.
4. $x=k\pi$, $0.869+k\pi$, $-0.869+k\pi$.
5. $\sin^2\frac{1}{2}A=\pm\frac{\sqrt{2}}{2}\sqrt{1-\cos A}$, $\cos^2\frac{1}{2}A=\pm\frac{\sqrt{2}}{2}\sqrt{1+\cos A}$.
6. $\sin 3A=3\sin A-4\sin^3 a$, $\cos 3A=4\cos^3 A-3\cos A$, $\tan 3A=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$.
7. Omit.
8. Omit.
9. (a) $\tan\theta$
   (b) $\frac{1}{2}$
   (c) 1
10. 1, $-1+\sqrt{2}$
11. $\frac{7\sqrt{65}}{65}$.

Exercise 20

1. (a) $\sqrt{53}, 15.9°, 74.1°$
   (b) $\sqrt{41}, 51.3°$
   (c) $\sqrt{34}, 59.0°$
2. 5.57
3. $-2\sin\alpha$
4. $[\frac{2}{37},\frac{2}{3}]$
5. 9, -17, 0.588, 0.983
6. the graph is as follows: period $\frac{2\pi}{3}$



7. π

Exercise 21

1. (a) 1
   (b) $-\frac{3}{4}$
2. $k = -1$, $m = -1$
3. $a_n = \cos\frac{1}{2}x\cos\frac{1}{4}x\cdots\cos^2\frac{1}{2^{n-1}}x$, $b_n = \cos\frac{1}{2}x\cos\frac{1}{4}x\cdots\cos\frac{1}{2^{n-1}}x$
4. omit
5. prove $\tan\frac{q+p}{2} = \tan\frac{r+s}{2}$

Exercise 22

1. (a) $3e^{3x+1}$
   (b) $\frac{1}{2}\cdot 2^{\sqrt{x}}(\ln 2)\cdot(x^{-\frac{1}{2}})$
   (c) $3(2+e^{1-x})^{-2}e^{1-x}$
   (d) $\frac{1}{3}e^{3x-x^2}(3-2x) - \frac{2}{3}e^{1-x-x^2}(-1-2x)$.
   (e) omit
2. Maximum, $(\frac{\ln 2}{2}, 4\sqrt{2})$
3. Minimum, $(-1, e^{-3})$
4. $x = 0$.
5. Omit.

Exercise 23

1. (a) $\frac{2x}{x^2+1}$
   (b) $\frac{2}{(2x-5)\ln 3}$
   (c) $\frac{3}{x} + \frac{8x}{x^2-1}$
   (d) $\frac{4}{4x+3} - \frac{3x^2}{x^3-1}$
   (e) $-\frac{1}{(x+1)(\ln(x+1))^2}$
   (f) $-\frac{2}{x(\ln x)^2}$.
2. $(1, \ln 2)$
3. $y = -x + 4$
4. $y - 2e = -2x$
5. $x = -3$. $x < -3$, then $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$, $x > -3$, then $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$.
6. (a) $(-\infty, \infty)$
   (b) $f^{-1}(x) = \frac{x+\sqrt{x^2+4}}{2}$.
   (c) $y - 1 = \frac{1}{2}(x - 0)$.

Exercise 24

1. (a) $2x\cos\left(x^2\right)$.
   (b) $-\frac{1}{2}\sin x(1+\cos x)^{-\frac{1}{2}}$.
   (c) $6\cos 2xe^{3\sin 2x}$.
   (d) $\tan x$.
   (e) $2e^{2x}\cos\left(e^{2x}\right)$.
   (f) $-\sin 2x$.
   (g) $\sin 2x$.
2. $y-\ln\sqrt{2}=x-\frac{\pi}{4}$
3. $y+\frac{1}{3}=-\frac{2}{3\sqrt{3}}(x-\frac{\pi}{6})$
4. $x=\frac{\pi}{6}$, max; $x=\frac{5}{6}\pi$, min.
5. $\cos 2x(1-\sin 2x)$.
6. omit, $-(\csc x-\sin x)^{-2}(-\csc x\cot x-\cos x)$.
7. Omit.
8. (a) $\frac{1}{2}\cos\frac{1}{2}x+\frac{1}{3}\sin\frac{1}{3}x$.
   (b) -1 and $\frac{1}{2}$
   (c) 12π
   (d) 12π
9. $\frac{1}{\sqrt{1-x^2}}$, $-\frac{1}{\sqrt{1-x^2}}$
10. (a) $-\frac{1}{2}x^{-\frac{1}{2}}\sin\sqrt{x}$
    (b) -$\frac{1}{2}\sin x(\cos x)^{-\frac{1}{2}}$
    (c) $2x\cos x^2$
    (d) $(-x^2)\cos\frac{1}{x}$.

Exercise 25

1. (a) $\sec^2(x)$.
   (b) $-\csc^2 x$
   (c) $x+2x\ln x$.
   (d) $2x\sec^2(x^2+1)$
   (e) $\dfrac{2xe^{2x}-(e^{2x}+1)}{x^2}$.
   (f) $(2x)\sin x^2+(x^2-2)(\cos x^2)(2x)$
2. Max $(-3,6e^{-3})$, Min $(1,-2e)$
3. omit
4. $(e,\frac{1}{3})$
5. $(1,\frac{1}{2})$
6. (a) $x=\frac{\pi}{4}$
   (b) max

7. $u_1'u_2u_3\cdots u_k + u_1u_2'\cdots u_k + \cdots + u_1u_2\cdots u_k'$
8. $\sum_{k=0}^{n} \binom{n}{k} u^{(n-k)}v^{(k)}$

Exercise 26

Omit

Exercise 27

1. $y - 4 = 3(x + 8)$
2. $x = a\cos t$, $y = a\sin t$. $AB = a$.
3. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{e^{-t}}{1-e^{-t}}$. $t = \ln 3 - \ln 2$.
4. $t = 3$, $(\frac{4}{3}, \frac{27}{e^3})$.
5. Omit.
6. (a) $\frac{5}{3}$
   (b) $-\frac{32}{27}$.
7. Omit.

Exercise 28

1. $-\frac{1}{\pi+2}$
2. $y - 2 = \frac{1}{3}(x - 1)$
3. Horizontal: $y = \pm 3\sqrt{3}$, vertical: $x = \pm 3$
4. $(-1, -2)$, max; $(0, 0)$, min.
5. $(\frac{8}{3}, -4)$
6. -1, 4.
7. Omit
8. $(2e, e^{\frac{1}{2}})$. $\ln y \neq 0$.
9. $(1, -4)$,min; $(-1, 4)$,max.
10. Omit.

Exercise 29

1. (a) $\frac{2}{3}\ln|3x - 1| + C$
   (b) $\frac{1}{2}e^{2x+3} + C$
   (c) $\frac{\sqrt{3}}{3}\tan^{-1}(\frac{2}{\sqrt{3}}x) + C$

(d) $-\frac{1}{2}\cos 2x + C$

(e) $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$

2. $x^4 - 3x^2 + x + 7$

Exercise 30

Omit

Exercise 31

1. $8\frac{5}{8} + 3\ln 2$
2. $\frac{1}{2} - \frac{1}{2e}$
3. 2
4. $\frac{\pi}{8} - \frac{1}{12}$
5. $\frac{\pi}{8}$
6. 0

Exercise 32

1. $1 - \frac{\pi}{4}$
2. $2\frac{11}{12} - \ln 6$
3. Omit
4. (a) $4e^{\frac{x}{2}} - 2x - 1$

   (b) min.
5. Omit
6. Omit
7. Omit
8. Omit
9. Omit

Exercise 33

1. If $p = 1$, infinite;

   if $p > 1$, $\int_0^1 \frac{1}{x^p}\mathrm{d}x$ infinite, $\int_1^\infty \frac{1}{x^p}\mathrm{d}x = \frac{1}{1-p}$;

   if $p < 1$, $\int_0^1 \frac{1}{x^p}\mathrm{d}x = \frac{1}{1-p}$, $\int_1^\infty \frac{1}{x^p}\mathrm{d}x$ infinite.
2. $\frac{1}{2}e$

3. Infinite
4. Omit

Exercise 34

1. (a) $\frac{287}{192}\pi + \pi \ln 4$
   (b) $\frac{\pi}{4}(e^2 - e^{-2})$
   (c) $\frac{\pi^2}{2}$
   (d) $\pi(\sqrt{3} - 1 - \frac{\pi}{12})$.
2. $\frac{1}{2}$
3. $2\pi^2 r a^2$
4. (a) $\pi(\sqrt{3} - \frac{\pi}{3})$
   (b) Omit.

Exercise 35

1. (a) $\displaystyle\int 1 + \frac{-2}{x-2} + \frac{-3}{x+3}\,\mathrm{d}x = x - 2\ln|x-2| - 3\ln|x+3| + C.$
   (b) $\displaystyle\int \frac{\frac{5}{3}}{x+4} + \frac{-\frac{3}{4}}{x+2} + \frac{\frac{1}{12}}{x-2}\,\mathrm{d}x = \frac{5}{3}\ln|x+4| - \frac{3}{4}\ln|x+2| + \frac{1}{12}\ln|x-2| + C.$
   (c) $\displaystyle\int \frac{1}{4}\frac{1}{1-x} + \frac{1}{4}\frac{1}{1+x} + \frac{1}{2}\frac{1}{1+x^2}\mathrm{d}x = -\frac{1}{4}\ln|1-x| + \frac{1}{4}\ln|1+x| + \frac{1}{2}\tan^{-1}x + C.$
   (d) $\displaystyle\int_{-\frac{1}{2}}^{1} \frac{1}{2}\frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{2}\frac{1}{x+3}\mathrm{d}x = \frac{1}{2}(\ln 8 - \ln 5).$
   (e) $\frac{3}{2}\ln 3 - \ln 4.$
2. $10\frac{5}{12}\pi$; 20.8333π

Exercise 36

1. $\mathrm{d}t = 2x\,\mathrm{d}x$
2. $\mathrm{d}x = t\,\mathrm{d}t$
3. $\mathrm{d}t = \cos x\,\mathrm{d}x$
4. $\mathrm{d}x = e^t\,\mathrm{d}t$
5. $\mathrm{d}x = \frac{1}{t+1}\,\mathrm{d}t$
6. $\mathrm{d}t = \frac{1}{2}\sec^2\frac{x}{2}\,\mathrm{d}x$
7. $\mathrm{d}t = \frac{2x}{1+x^4}\,\mathrm{d}x$
8. $\mathrm{d}t = -\tan x\,\mathrm{d}x$

Exercise 37

1. $e^{\sin x} + C$
2. $-\frac{1}{6}(3 - 2x^2)^{\frac{3}{2}} + C$
3. $\frac{1}{3}\ln|x^3 - 1| + C$
4. $\frac{\pi}{8}$
5. $\frac{7}{24}$
6. $\frac{3}{14}(x^2 - 1)^7 + \frac{1}{4}(x^2 - 1)^6 + C$
7. $x + 4\sqrt{x} + 4\ln|\sqrt{x} - 1| + C$
8. $\frac{27}{140}$
9. $\frac{1696}{2835}$
10. $\frac{3}{8}$
11. $\frac{2}{3}\pi$
12. π
13. $2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\ln|1 + x^{\frac{1}{6}}| + C$
14. $\ln(\sec x + \tan x) + C$
15. $\frac{1}{3}\ln|1 + x| - \frac{1}{3}\left(\frac{1}{2}\ln|x^2 - x + 1| - \sqrt{3}\tan^{-1}\left(\frac{2}{\sqrt{3}}(x - \frac{1}{2})\right)\right) + C$

Exercise 38

1. $\frac{1}{3}x\mathrm{e}^{3x} - \frac{1}{9}\mathrm{e}^{x} + C$
2. $\frac{1}{5}x^5\ln 3x - \frac{1}{25}x^5 + C$
3. $\frac{-x\ln x - 1}{x} + C$
4. $\frac{\sqrt{2}}{2}(4 - \pi)$
5. $\frac{\pi^2}{4}$
6. $-\frac{1}{2}x^2\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$
7. $\frac{4}{25}\mathrm{e}^{3x}\sin 4x + \frac{3}{25}\mathrm{e}^{3x}\cos 4x + C$
8. $\frac{3}{10}(\mathrm{e}^{-\pi} - \mathrm{e}^{\pi})$
9. $\frac{\pi}{4} - \frac{1}{2}\ln 2$
10. $\frac{1}{3}x^3\ln^2 x - \frac{2}{9}x^3\ln x + \frac{2}{27}x^3 + C$
11. $\frac{1}{2}\mathrm{e}^{x^2}(x^2 - 1) + C$
12. $2\mathrm{e}^{\sin x}(\sin x - 1) + C$
13. $4\left(\sqrt{2} - 1\right)$

Exercise 39

1. $-\frac{\sqrt{1+x^2}}{x}+C$
2. $\frac{1}{11}\tan^{11}x+C$
3. $-2\cos\sqrt{x}+C$
4. $\frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right|+C$
5. $\frac{3}{2}(\sin x-\cos x)^{\frac{3}{2}}+C$
6. $-(\arcsin x)^{-1}+C$
7. $-\ln\left|\cos\sqrt{1+x^2}\right|+C$
8. $\frac{1}{2}\left(\ln\left|\sqrt{1+e^{2x}}-1\right|-\ln\left|\sqrt{1+e^{2x}}+1\right|\right)+C$
9. $-\frac{1}{3}\left(\frac{x}{\sqrt{1+x^2}}\right)^{-3}+\left(\frac{x}{\sqrt{1+x^2}}\right)^{-1}+C$
10. $-\ln\left|1+\sqrt{\frac{x-1}{x+1}}\right|+\ln\left|1-\sqrt{\frac{x-1}{x+1}}\right|-\left(1+\sqrt{\frac{x-1}{x+1}}\right)^{-1}+\left(1-\sqrt{\frac{x-1}{x+1}}\right)^{-1}+C$
11. $\frac{x}{\sqrt{1-x^2}}+C$
12. $\frac{1}{2}\arcsin x+\frac{1}{2}\ln\left|x+\sqrt{1-x^2}\right|+C$
13. $\sqrt{2x}-\ln\left|1+\sqrt{2x}\right|+C$
14. $\operatorname{arcsec}x+C$
15. $(\arctan\sqrt{x})^2+C$
16. $\frac{1}{4}\ln\left|\frac{\sqrt[4]{1+x^4}-1}{\sqrt[4]{1+x^4}+1}\right|+\frac{1}{2}\arctan\sqrt[4]{1+x^4}+C$
17. $-(x\ln x)^{-1}+C$
18. $2\left(\sqrt{x+1}\mathrm{e}^{\sqrt{x+1}}-\mathrm{e}^{\sqrt{x+1}}\right)+C$
19. $x(-\cot x)+\ln|\sin x|+C$
20. $x\arcsin x+\sqrt{1-x^2}+C$
21. $\arctan\left(\frac{1}{3}x^3\right)-\frac{1}{3}\left(\frac{1}{2}x^2-\frac{1}{2}\ln\left|1+x^2\right|+C\right)$
22. $x\ln^2x-2x\ln x+2x+C$
23. $x\tan x+\ln|\cos x|-\frac{x^2}{2}+C$
24. $-x^{-1}\ln^3x-3x^{-1}\ln^2x-6x^{-1}\ln x-6x^{-1}+C$
25. $\frac{1}{2}(\sin(\ln x)+\cos(\ln x))+C$
26. $x(\arcsin x)^2+2(\arcsin x)\left(1-x^2\right)^{\frac{1}{2}}+2x+C$
27. $\mathrm{e}^{\sqrt{x}}(2x-4\sqrt{x}+4)+C$
28. $x\ln\left(x+\sqrt{1+x^2}\right)-\left(1+x^2\right)^{\frac{1}{2}}+C$
29. *Omit*
30. *Omit*
31. *Omit*

32. *Omit*
33. *Omit*
34. *Omit*
35. *Omit*
36. *Omit*
37. *Omit*
38. *Omit*

Exercise 40

1. *Omit*
2. *Omit*
3. *Omit*
4. *Omit*
5. *Omit*
6. *Omit*
7. *Omit*
8. *Omit*
9. *Omit*
10. *Omit*
11. *Omit*
12. *Omit*
13. *Omit*

Exercise 41

1. Omit
2. $-2, 1, 16$
3. 1.466
4. 7.725

Exercise 42

1. 1.17
2. 1.37

3. 2.38
4. 0.739
5. 0.79

Exercise 43

1. (a) Omit
   (b) $3x^2 > 1$ for $x \in (1, 2)$
   (c) $|\frac{1}{3}(x+1)^{-\frac{2}{3}}| < 1$ for $x \in (1, 2)$
2. (a) 3
   (b) 3.25
   (c) $x = -\sqrt{e^x - 5x + 1}$, -5.19; $x = \frac{e^x - x^2 + 1}{5}$, 0.48.

Exercise 44

1. 6.2417.
2. 1.95.
3. 0.7828, 0.7842, 0.7847. 0.7854
4. 7.500, 7.306.

Exercise 45
Omit

Exercise 46

1. (a) $y = \frac{1}{2}x^2 + x + \ln|x - 1| + C$
   (b) $x = -\frac{\ln y}{y} - \frac{1}{y} + C$
   (c) $\frac{1}{2}y^2 = \frac{1}{3}x^3 + C$
   (d) $u = \frac{C}{\cos t}$
   (e) $\ln(1 + y^2) = xe^x - e^x + C$
   (f) $e^y = x \ln x + C$
   (g) $\ln|1 - w| - \frac{1}{w-1} = -t \cos t + \sin t + C$
2. $\ln y = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + \frac{1}{4}$
3. $y = \frac{1}{2}(1 + x^2)$
4. $y = (\sqrt{x} + 1)^2$
5. (a) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 0.3\sqrt{x} - 0.02x$

(b) $x = \left(\dfrac{0.3 - e^{-100(t+C)}}{0.02} \right)^2$

(c) $x \longrightarrow 225$

6. (a) $y = x - 1$

   (b) $y = e^{-\int p(x)\,\mathrm{d}x} \left(\int e^{\int p(x)\,\mathrm{d}x} q(x)\,\mathrm{d}x \right)$

Exercise 47

Omit

Exercise 48

Omit

Exercise 49

1. Opposite
2. $\overrightarrow{FO}, \overrightarrow{OC}, \overrightarrow{ED}$

Exercise 50

1. $\begin{pmatrix} 1 \\ a \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \end{pmatrix}$.
2. $E(1,1)$, $F(4,-3)$
3. $\vec{x} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$
4. $\overrightarrow{GY} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$

Exercise 51

1. $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
2. $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$
3. (a) $\overrightarrow{DB}$

   (b) $\vec{e}$

   (c) $\overrightarrow{BA}$

   (d) $\overrightarrow{DA}$

   (e) $\vec{b}$

Exercise 52

Omit

Exercise 53

1. Omit
2. $\vec{r} = \begin{pmatrix} \frac{5}{\sqrt{61}} \\ \frac{6}{\sqrt{61}} \end{pmatrix}$
3. $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\frac{1}{2}\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $-\overrightarrow{BC} = 6\mathbf{i} - \mathbf{j} - \mathbf{k}$
4. $E(10, 2, -3)$
5. $\overrightarrow{AB} = \overrightarrow{DC} = \begin{pmatrix} 1 \\ 1 \\ -6 \end{pmatrix}$, parallelogram
6. 0 or $-\frac{1}{3}$
7. Omit
8. $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, distance between two points.

Exercise 54

Omit

Exercise 55

1. 14, 1
2. Omit
3. NO.
4. Omit
5. NO.

Exercise 56

1. $\vec{a}$ and $\vec{d}$; $\vec{b}$ and $\vec{c}$
2. Omit
3. 98.1°, 151.9°, 166.0°, 122.5°, 27.2°, 161.9°, invalid, invalid.
4. 24.3°, 8.12
5. 45°
6. $\arccos\left(\frac{1}{3}\right)$
7. $\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$ or $-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$

8. $x = -\frac{1}{2}$, $y = \pm 1$.
9. Omit
10. Omit

Exercise 57

Omit

Exercise 58

Omit

Exercise 59

1. (a) Skew
   (b) intersect $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$
   (c) parallel
   (d) the same line
   (e) Skew
2. (a) $\cos POQ = \dfrac{-2p - 9 - q}{\sqrt{14}\sqrt{p^2 + q^2 + 9}}$
   (b) $p = -\frac{11}{5}$, $q = -\frac{23}{5}$, or $p = -5$, $q = -1$
   (c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
   (d) $\begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix}$, $\sqrt{14}$
3. (a) 79.9°
   (b) $\frac{1}{\sqrt{29}} \begin{pmatrix} 2 \\ -4 \\ -3 \end{pmatrix}$
   (c) 7 or 1
   (d) Skew
4. (a) Omit
   (b) $\begin{pmatrix} -\frac{2}{3} \\ \frac{11}{3} \\ \frac{1}{3} \end{pmatrix}$
   (c) B is further.
5. (a) 4.09
   (b) 1.90
   (c) 3.03

Exercise 60

1. $5 + 2\text{i},\ \frac{1}{5} + \frac{7}{5}\text{i},\ \frac{3}{5} + \frac{1}{5}\text{i},\ \frac{1}{17}(25 - 15\text{i})$
2. $3, -\frac{1}{2}, -5 + \sqrt{2}\text{i}, -1, 1.$
3. FTTTFFT
4. $x = 1,\ y = 1$
5. $5a + 4b + 3c = 0$
6. (a) $1 + 2\text{i}$ or $-1 - 2\text{i}$
   (b) $3 + 2\text{i}$ or $-3 - 2\text{i}$.
7. $\pm(2 + \text{i}),\ \pm(1 - 2\text{i}).$

Exercise 61

1. Omit
2. $-5 - 10\text{i}, 6 - 4\text{i}, \frac{3}{8} + \frac{5}{8}\text{i}, 6 - 4\text{i}, \frac{3}{8} + \frac{5}{8}\text{i}.$
3. $\frac{\sqrt{40}}{\sqrt{13}+\sqrt{17}}, -2 - 4\text{i}, \frac{2197}{289}, 10, -\frac{70}{17} - \frac{212}{17}\text{i}.$

Exercise 62

1. Omit
2. $\sqrt{41} + 3$
3. $\frac{2}{5}\sqrt{5}$
4. $\frac{7}{2}\sqrt{2} - 1$
5. Omit
6. Omit
7. 2 and 8
8. $(\frac{41}{14}, \frac{27}{14})$.
9. $x = \frac{(y+1)^2}{4} + 1.$

Exercise 63
Omit

Exercise 64

1. (a) $2\sqrt{6}\text{i}$
   (b) $2 - \sqrt{3} - 2\text{i}$
   (c) $0.885 + 4\text{i}$ or $-0.787 + 1.1015\text{i}$

(d) $-2\mathrm{i}$.

2. Omit

3. (a) $\frac{1-\sqrt{17}}{2}$
   (b) 0.723
   (c) 1.14

Exercise 65

1. $zw = 4\sqrt{2}\mathrm{e}^{\mathrm{i}(-\frac{11}{12}\pi)}$, $\frac{z^*}{w} = \sqrt{2}\mathrm{e}^{\mathrm{i}\frac{11}{12}\pi}$.
2. $\frac{5}{12}\pi$
3. (a) $2\mathrm{e}^{\mathrm{i}\frac{7}{12}\pi}$.
   (b) $4\mathrm{e}^{\mathrm{i}\frac{11}{12}\pi}$.
   (c) 8.
   (d) $2\mathrm{e}^{\mathrm{i}-\frac{1}{12}\pi}$.
   (e) $\mathrm{e}^{\mathrm{i}-\frac{1}{3}\pi}$.
   (f) $32\mathrm{e}^{\mathrm{i}-\frac{3}{4}\pi}$.
4. Omit
5. $-\sqrt{3}+\mathrm{i}$
6. $(3+2\sqrt{3}, 1+2\sqrt{3})$, $(3-2\sqrt{3}, 1-2\sqrt{3})$
7. $(2,-1)$, $(4,1)$,$(6,1)$,$(2,-3)$,$(4,3)$,$(0,-1)$
8. Omit.

Exercise 66
Omit